NORTH SYDNEY GIRLS HIGH SCHOOL

2019 | **HSC Trial Examination**

Mathematics

General Instructions

- Reading Time – 5 minutes
- Working Time – 3 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Total marks – 100

Section I – 10 marks (pages 2 – 6)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 7 – 15)

- Attempt Questions 11 – 16
- Allow about 2 hours 45 minutes for this section

NAME:______________________________ **TEACHER:**______________________________

STUDENT NUMBER:

Question	1-10	11	12	13	14	15	16	Total
Mark	/10	/15	/15	/15	/15	/15	/15	/100

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1-10.

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1 What is the value of $1.15 \div 15$ correct to three significant figures?

A. 0.076

B. 0.077

C. 0.0766

D. 0.0767

2 What is the solution to $x^2 - 5x - 6 > 0$?

A. $x < -6$ or $x > 1$

B. $x < -1$ or $x > 6$

C. $-6 < x < 1$

D. $-1 < x < 6$

3 If $2x + 3y + 1 = 0$ is perpendicular to $5x + ky - 1 = 0$, what is the value of k?

A. $\frac{15}{2}$

B. $\frac{10}{3}$

C. $-\frac{15}{2}$

D. $-\frac{10}{3}$

4 What is the shortest distance from $(-2,3)$ to the line $y = 3x - 5$?

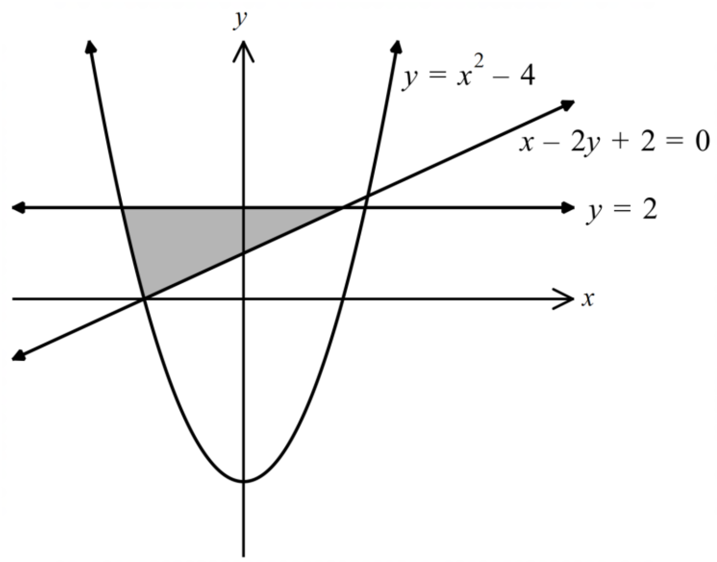
A. $\dfrac{14}{\sqrt{10}}$

B. $\dfrac{14}{\sqrt{13}}$

C. $\dfrac{8}{\sqrt{10}}$

D. $\dfrac{8}{\sqrt{13}}$

5 Which of the following set of inequalities represents the shaded region?



A. $y \ge x^2 - 4$ and $y \le 2$ and $x - 2y + 2 \le 0$

B. $y \le x^2 - 4$ and $y \le 2$ and $x - 2y + 2 \le 0$

C. $y \ge x^2 - 4$ and $y \le 2$ and $x - 2y + 2 \ge 0$

D. $y \le x^2 - 4$ and $y \le 2$ and $x - 2y + 2 \ge 0$

6 What are the coordinates of the focus of the parabola $y^2 = 4 - x$?

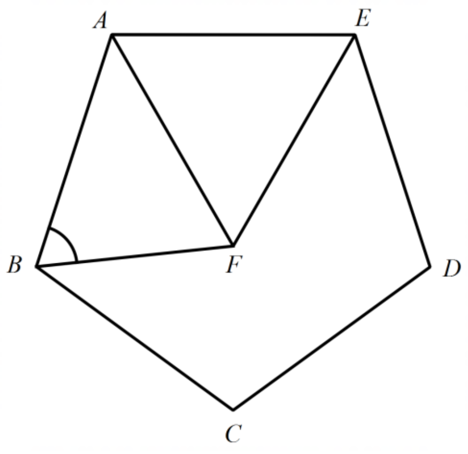
A. $\left(0, \dfrac{15}{4}\right)$

B. $\left(0, \dfrac{17}{4}\right)$

C. $\left(\dfrac{15}{4}, 0\right)$

D. $\left(\dfrac{17}{4}, 0\right)$

7 *ABCDE* is a regular pentagon and *AEF* is an equilateral triangle. What is the size of $\angle ABF$?



A. 48°

B. 60°

C. 66°

D. 108°

8 Given that $0 < x < y < \dfrac{\pi}{4}$, which of the following statements are true?

I $\sin x < \sin y$
II $\cos x < \cos y$
III $\sin x < \cos y$

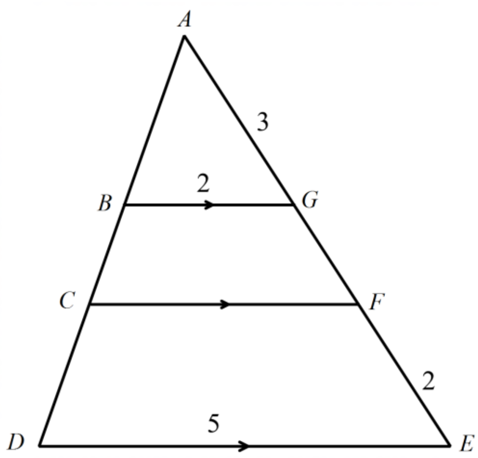
A. I only

B. II only

C. I and II only

D. I and III only

9 In the diagram below, $BG = 2$, $DE = 5$, $AG = 3$ and $EF = 2$. What is the length of *CF*?



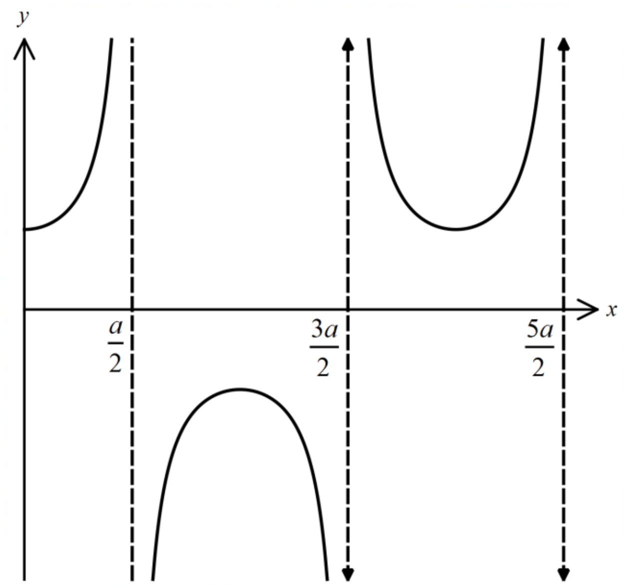
A. $\dfrac{15}{2}$

B. $\dfrac{11}{3}$

C. $\dfrac{5}{3}$

D. 3

10 Given, the graph below, where $a > 0$, which of the following could be the equation of the graph ?



A. $y = \operatorname{cosec}\left(\frac{\pi}{a}\left(x + \frac{a}{2}\right)\right)$

B. $y = \operatorname{cosec}\left(\frac{2\pi}{a}\left(x + \frac{a}{2}\right)\right)$

C. $y = \operatorname{cosec}\left(\frac{\pi}{a}\left(x + \frac{a}{4}\right)\right)$

D. $y = \operatorname{cosec}\left(\frac{2\pi}{a}\left(x + \frac{a}{4}\right)\right)$

Section II

90 marks
Attempt Questions 11-16
Allow about 2 hours 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks**)** Use a SEPARATE writing booklet

(a) Solve $|4x-3|>5$. **2**

(b) Simplify $\dfrac{x-3}{x^2+4x+3}-\dfrac{x}{x+3}$. **2**

(c) Given that $f(x)=16x+\dfrac{1}{x}$ find the values of x for which $f'(x)=0$. **2**

(d) Differentiate:

(i) $x^3\log_e x$ **2**

(ii) $\dfrac{e^x}{\cos x}$ **2**

(e) (i) Show that if $2\operatorname{cosec}^2 x=5-5\cot x$ then $\tan x=2$ or $\tan x=-\dfrac{1}{3}$. **3**

(ii) Hence, or otherwise solve $2\operatorname{cosec}^2 x=5-5\cot x$ in the domain $-\pi\le x\le\pi$ giving your answers correct to 1 decimal place. **2**

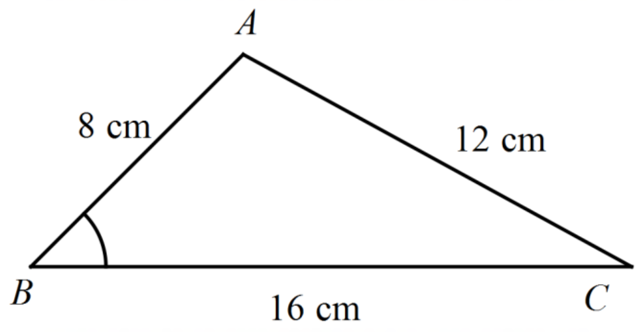
Question 12 (15 marks)

(a) The volume V m^3 of water in a tank at time t seconds is given by **2**

$$V = \frac{1}{2}t^4 - 2t^3 + 3t + 5, \quad \text{for } t \geq 0$$

Find the rate at which the volume is changing after 3 seconds.

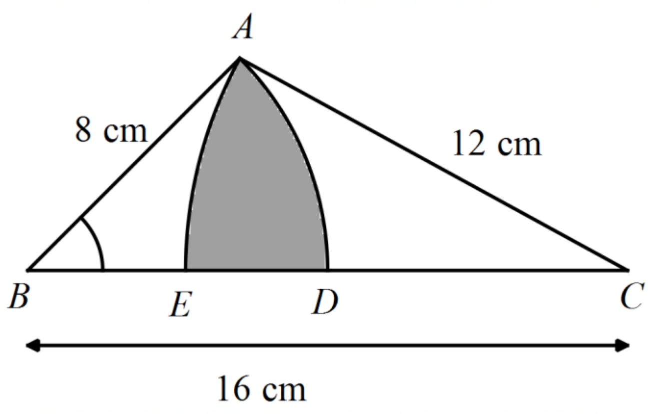
(b) The triangle ABC shown below has sides $AB = 8$ cm, $BC = 16$ cm and $AC = 12$ cm respectively.



NOT TO SCALE

(i) Show that $\angle ABC = 46°34'$ correct to the nearest minute. **2**

Circular arcs are now drawn from B and C through A to meet BC at D and E respectively.



NOT TO SCALE

(ii) Show that the length of arc AD is 6.5 cm. **1**

(iii) Given that $\angle ACB = 0.505$ (3 s.f.), find the area of sector ACE and sector ABD. **2**

(iv) Hence, or otherwise, find the shaded area. **1**

Question 12 continues on Page 9

(c) Consider the arithmetic sequence 1000, 994, 988, 982,…. **3**

Find the maximum value of S_n, the sum of the first n terms.

(d) The rate at which the concentration of a drug in a patient's bloodstream is decreasing is proportional to the concentration of the drug C present in the bloodstream at that time. The time is measured in hours, from the time the drug was administered and C is the concentration of the drug in micrograms per litre.

The concentration of the drug can be described by the model $C = C_0 e^{-kt}$, where C_0 is the initial concentration and k is a positive constant based on the age and weight of the patient.

(i) In a particular patient, it was observed that the drug concentration was reduced to 40% of its initial value after two hours. **2**

Show that the exact value of k is $\frac{1}{2}\log_e\left(\frac{5}{2}\right)$.

(ii) The first dose was administered to the patient at midnight. **2**
The next dose is to be given when the concentration drops to 10% of its initial value. At what time should the patient be given the next dose?

End of Question 12

Question 13 (15 marks)

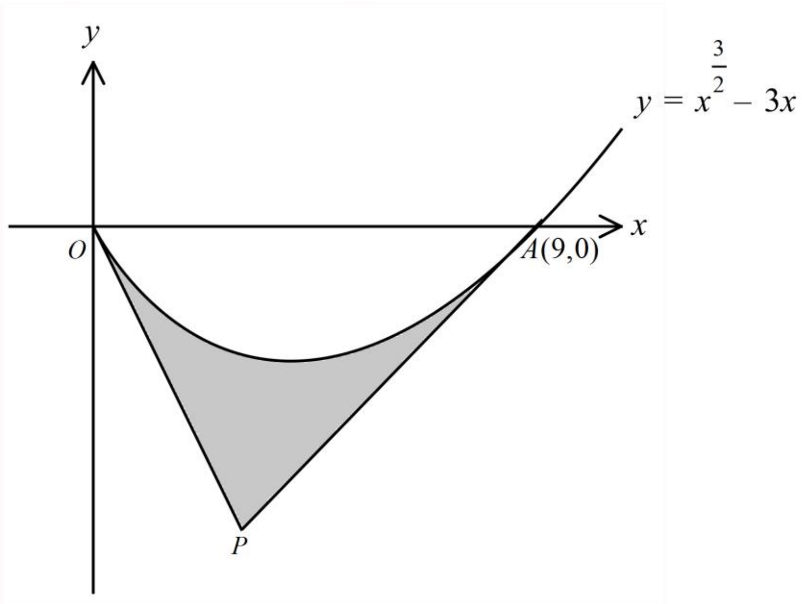
(a) Find the values of k for which the function $f(x) = 2x^3 - kx^2 + x + 1$ is increasing for all values of x. **3**

(b) The limiting sum of an infinite geometric series is five times the first term.

(i) Show that the common ratio of the series is 0.8. **2**

(ii) If the first term is 20, find the least value of n for which the n^{th} term is less than 1. **2**

(c) A portion of the curve $y = x^{\frac{3}{2}} - 3x$ for $x \geq 0$ is shown below.
The curve passes through the origin and cuts the positive x-axis again at $A(9,0)$.



(i) Find $\frac{dy}{dx}$. **1**

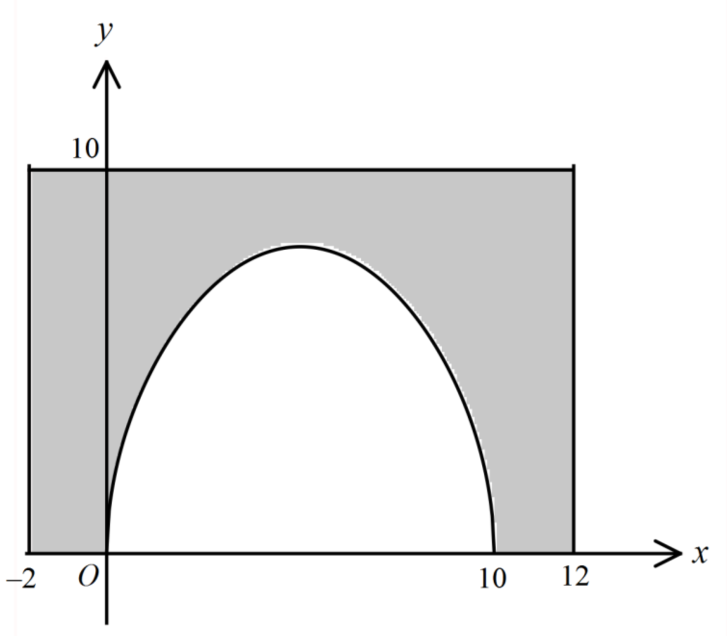
(ii) Show that the tangent to the curve at A is $3x - 2y - 27 = 0$. **2**

(iii) It is given that the equation of the tangent to the curve at O is $3x + y = 0$. Find the coordinates of the point P where the two tangents intersect. **2**

(iv) Find the area of the shaded region enclosed between the curve and the two tangents. **3**

Question 14 (15 marks)

(a) The cross-section of a tunnel with its concrete surround is shown below.



The concrete surround is represented by the shaded region bounded by the curve, the lines $x = -2,\ x = 12,\ y = 10$ and the x-axis. The units on both axes are in metres.

The height of the tunnel is measured at regular intervals. The results are shown in the table below correct to two decimal places.

x	0	2	4	6	8	10
y	0	6.13	7.80	7.80	6.13	0

(i) The area of the cross-section of the tunnel is given by $\int_0^{10} y\,dx$. **2**

Estimate this area using the Trapezoidal Rule with the given function values.

(ii) Hence, deduce an estimate for the cross-section area of the concrete surround. **1**

(iii) State with justification whether the area of the concrete cross-section found in (ii) is an over-estimate or an underestimate. **1**

Question 14 continues on Page 12

(b) Consider the function $f(x) = x^4 - 3x^3 + 3$.

(i) Find the stationary points on the curve and determine their nature. **3**

(ii) Find the x-coordinates of any points of inflexion. **2**

(iii) Sketch the curve for $-1 \le x \le 3$ showing all the information from parts (i) and (ii). You are not required to find the x-intercepts. **2**

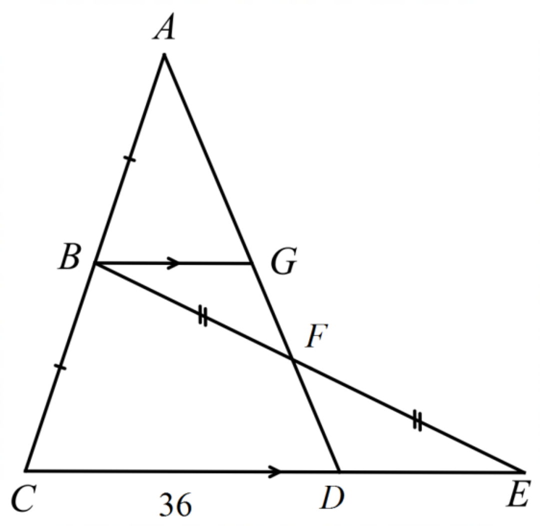
(c) The region enclosed between the curve $y = e^x + 3$ and the x-axis from $x = 0$ to $x = a$ is rotated about the x-axis to form a solid of revolution. **4**

If the volume of this solid is $\pi(9a + 16)$ units^3, find the exact value of a.

End of Question 14

Question 15 (15 marks)

(a) In the diagram below, $AB = BC$ and $EF = FB$ as shown. BG is parallel to CE and $CD = 36$ cm.



(i) Prove that ΔDEF and ΔBGF are congruent. **3**

(ii) Hence, or otherwise, find the length of DE. **2**

(iii) Given that $\angle CAD = \angle GBF$, prove that ΔACD and ΔEBC are similar. **2**

(iv) Deduce the exact length of AC. **2**

(b) The motion of a particle moving in a straight line is described by $v = \dfrac{6}{t+3}$, where v is the velocity in metres/second and t is the time in seconds. Initially the particle is at the origin.

(i) What is the acceleration of the particle 2 seconds after the start of the motion? **2**

(ii) Find the displacement, x, as a function of time. **2**

(iii) Briefly describe the motion of the particle. **2**

Question 16 (15 marks)

(a) (i) Given that $f(x) = x^4 + 2x$, find $f'(x)$. **1**

(ii) Hence, or otherwise, find $\int_1^3 \frac{2x^3 + 1}{x^4 + 2x}\,dx$. **2**

(b) Selina borrowed an amount \$$P$ at a rate of 3.6% per annum compounded monthly on a reducing balance. At the end of each month, she repays an amount of \$$M$.

(i) Show that the amount owing at the end of the second month is **1**

$A_2 = P(1.003)^2 - M(1.003) - M$.

(ii) It is now given that the loan amount is \$50 000 and her monthly repayment is \$350. Starting with your result in (i), show that the amount owing after one year is \$ 47 560. **2**

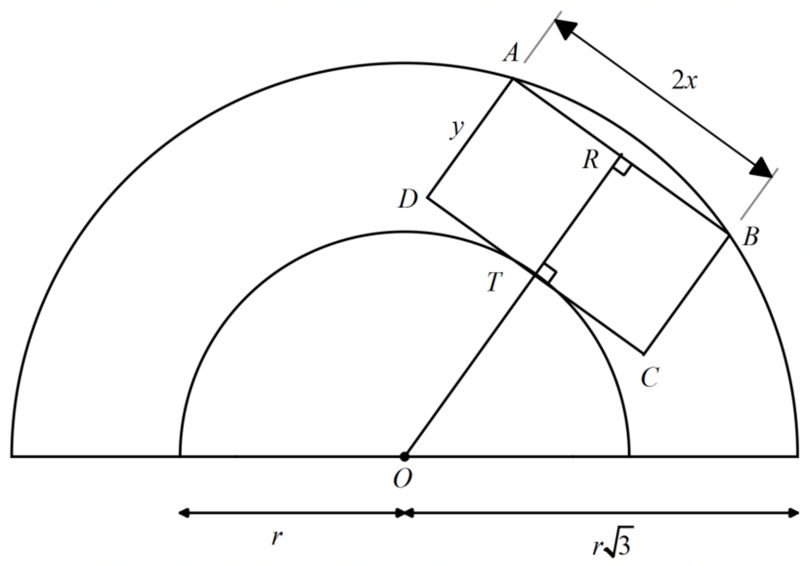
(iii) Find how many years it will take Selina to repay the loan fully. **3**

Question 16 continues on Page 15

(c) A rectangular box is required to be moved along a corridor bounded by curved vertical walls. A cross-section of the corridor is shown below.

The curved walls are two concentric semicircles centred at O, of radii r and $r\sqrt{3}$ respectively.

The rectangular box $ABCD$ has length $2x$ and width y as shown. CD is tangential to the inner wall at T, where T is the midpoint of CD. A and B are in contact with the outer wall. OTR is perpendicular to CD and AB.



(i) Show that $x^2 = 2r^2 - 2ry - y^2$. **2**

(ii) Show that the area of $ABCD$ is maximum when $y = \dfrac{r}{2}$. **4**

End of paper

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1-10.

---

1 What is the value of $1.15 \div 15$ correct to three significant figures?

A. 0.076

B. 0.077

C. 0.0766

D. 0.0767

2 What is the solution to $x^2 - 5x - 6 > 0$?

A. $x < -6$ or $x > 1$

B. $x < -1$ or $x > 6$

C. $-6 < x < 1$

D. $-1 < x < 6$

3 If $2x + 3y + 1 = 0$ is perpendicular to $5x + ky - 1 = 0$, what is the value of *k*?

A. $\frac{15}{2}$

B. $\frac{10}{3}$

C. $-\frac{15}{2}$

D. $-\frac{10}{3}$

$$-\frac{2}{3} \times -\frac{5}{k} = -1$$

so, $k = -\frac{10}{3}$

4 What is the shortest distance from $(-2,3)$ to the line $y = 3x - 5$?

A. $\frac{14}{\sqrt{10}}$

B. $\frac{14}{\sqrt{13}}$

C. $\frac{8}{\sqrt{10}}$

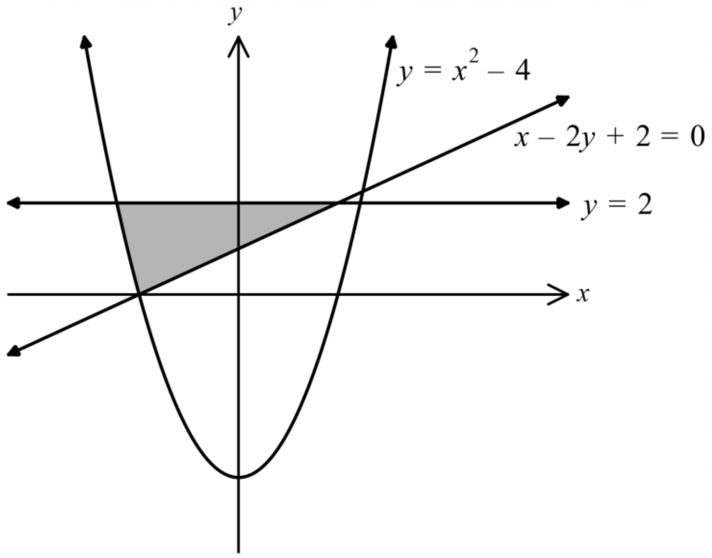
D. $\frac{8}{\sqrt{13}}$

Using the perpendicular distance formula

$3x - y - 5 = 0$

$$d = \frac{|3(-2) - 3 - 5|}{\sqrt{3^2 + (-1)^2}} = \frac{14}{\sqrt{10}}$$

5 Which of the following set of inequalities represents the shaded region?



A. $y \ge x^2 - 4$ and $y \le 2$ and $x - 2y + 2 \le 0$

B. $y \le x^2 - 4$ and $y \le 2$ and $x - 2y + 2 \le 0$

C. $y \ge x^2 - 4$ and $y \le 2$ and $x - 2y + 2 \ge 0$

D. $y \le x^2 - 4$ and $y \le 2$ and $x - 2y + 2 \ge 0$

Test (0,0)

6 What are the coordinates of the focus of the parabola $y^2 = 4 - x$?

A. $\left(0, \frac{15}{4}\right)$

B. $\left(0, \frac{17}{4}\right)$

C. $\left(\frac{15}{4}, 0\right)$

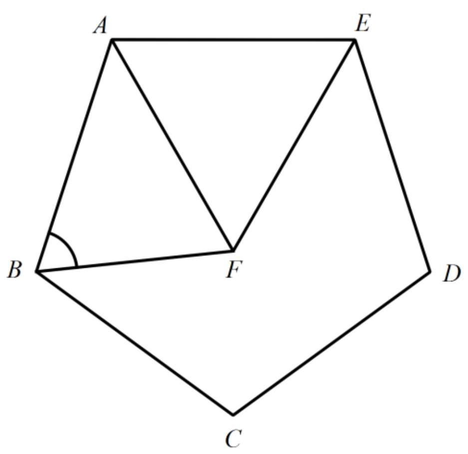
D. $\left(\frac{17}{4}, 0\right)$

$y^2 = -4\left(\frac{1}{4}\right)(x-4)$

$V = (4,0)$ and $a = \frac{1}{4}$ and parabola faces left

Focus is $\frac{1}{4}$ units to the left of $(4,0)$

7 *ABCDE* is a regular pentagon and *AEF* is an equilateral triangle. What is the size of $\angle ABF$?



A. 48°

B. 60°

C. 66°

D. 108°

$\angle EAF = 60°$ equilateral triangle

$\angle EAB = 108°$ regular pentagon

$\angle BAF = 48°$ and $\angle AFB = \angle ABF$ opposite equal sides

$\angle ABF = 66°$ using angle sum ΔABF

8 Given that $0 < x < y < \frac{\pi}{4}$, which of the following statements are true?

I	$\sin x < \sin y$
II	$\cos x < \cos y$
III	$\sin x < \cos y$

A. I only

B. II only

C. I and II only

D. I and III only

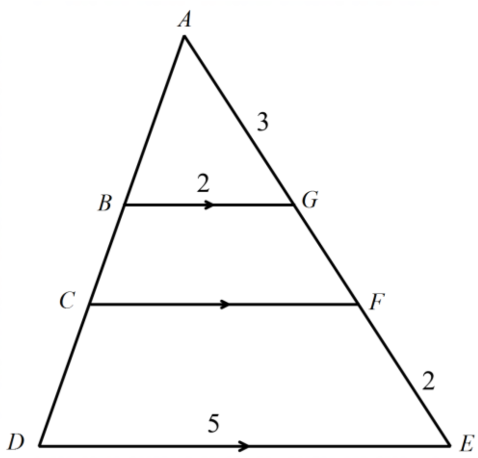
sin$x$ is increasing in the first quadrant so I is true

cos$x$ is decreasing in first quadrant so II is false

graph of sin$x$ lies below graph of cos$x$ for

$0 < x < \frac{\pi}{4}$, so III is true

9 In the diagram below, $BG = 2$, $DE = 5$, $AG = 3$ and $EF = 2$. What is the length of CF?



A. $\frac{15}{2}$

B. $\frac{11}{3}$

C. $\frac{5}{3}$

D. 3

Let $FG = x$

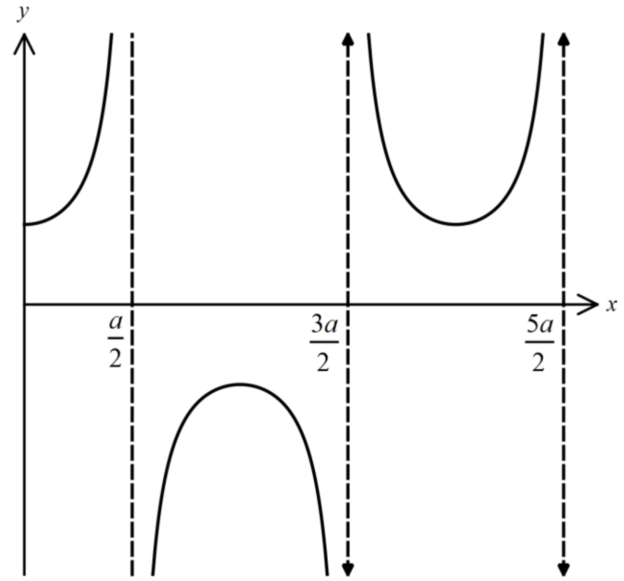
$\frac{3}{5+x} = \frac{2}{5}$ ratios of sides in similar triangles

$x = \frac{5}{2}$

$\frac{3}{3+x} = \frac{2}{CF}$ ratios of sides in similar triangles

$CF = \frac{11}{3}$

10 Given, the graph below, where $a > 0$, which of the following could be the equation of the graph ?



A. $y = \operatorname{cosec}\left(\frac{\pi}{a}\left(x + \frac{a}{2}\right)\right)$

B. $y = \operatorname{cosec}\left(\frac{2\pi}{a}\left(x + \frac{a}{2}\right)\right)$

C. $y = \operatorname{cosec}\left(\frac{\pi}{a}\left(x + \frac{a}{4}\right)\right)$

D. $y = \operatorname{cosec}\left(\frac{2\pi}{a}\left(x + \frac{a}{4}\right)\right)$

One cycle is $2a$. So $\frac{2\pi}{n} = 2a$ so $n = \frac{\pi}{a}$.

So A or C

Shifted left by quarter cycle ie by $\frac{a}{2}$ so $x + \frac{a}{2}$

Therefore A

Section II

90 marks
Attempt Questions 11-16
Allow about 2 hours 45 minutes for this section

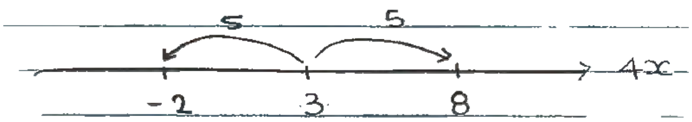
Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks**)** Use a SEPARATE writing booklet

(a)	Solve $\lvert 4x-3\rvert > 5$.	**2**

$|4x-3| > 5$



$4x < -2$ or $4x > 8$

$x < -\frac{1}{2}$ or $x > 4$

OR

Case 1: $x \geq \frac{3}{4}$ or Case 2: $x < \frac{3}{4}$

$4x - 3 > 5$

$4x$

$x > 2$

$-(4x-3) > 5$

$4x - 3 < -5$

$4x < -2$

$x < -\frac{1}{2}$

$\therefore\ x < -\frac{1}{2}$ or $x > 2$

(b)	Simplify $\dfrac{x-3}{x^2+4x+3} - \dfrac{x}{x+3}$.	**2**

$$\frac{x-3}{x^2+4x+3} - \frac{x}{x+3}$$

$$= \frac{x-3}{(x+1)(x+3)} - \frac{x}{x+3}$$

$$= \frac{(x-3) - x(x+1)}{(x+1)(x+3)}$$

$$= \frac{\cancel{x} - 3 - x^2 - \cancel{x}}{(x+1)(x+3)}$$

$$= -\frac{x^2+3}{(x+1)(x+3)}$$

Several students lost a mark here for getting an incorrect sign when expanding and simplifying the numerator, in particular with the second term.

(c)	Given that $f(x) = 16x + \frac{1}{x}$ find the values of x for which $f'(x) = 0$.	**2**

$f(x) = 16x + \frac{1}{x} = 16x + x^{-1}$

$f'(x) = 16 - x^{-2}$

$= 16 - \frac{1}{x^2}$

$f'(x) = 0 \Rightarrow 16 - \frac{1}{x^2} = 0$

$\frac{1}{x^2} = 16$

$x^2 = \frac{1}{16}$

$x = \pm \frac{1}{4}$

(d)	Differentiate:	
	(i) $x^3 \log_e x$	**2**

$y = x^3 \log x$

$y' = 3x^2 \log x + x^3 \times \frac{1}{x}$ Product Rule

$= 3x^2 \log x + x^2$

$= x^2 (3 \log x + 1)$

	(ii) $\frac{e^x}{\cos x}$	**2**

$y = \frac{e^x}{\cos x}$

$y' = \frac{e^x \cos x - e^x(-\sin x)}{(\cos x)^2}$ Quotient Rule

$= \frac{e^x \cos x + e^x \sin x}{\cos^2 x}$

$= \frac{e^x(\cos x + \sin x)}{\cos^2 x}$

(e) (i)	Show that if $2\,\text{cosec}^2 x = 5 - 5\cot x$ then $\tan x = 2$ or $\tan x = -\frac{1}{3}$.	**3**

$$2\,\text{cosec}^2 x = 5 - 5\cot x$$
$$2(1+\cot^2 x) = 5 - 5\cot x$$
$$2 + 2\cot^2 x - 5 + 5\cot x = 0$$
$$2\cot^2 x + 5\cot x - 3 = 0$$

Let $u = \cot x$

$$2u^2 + 5u - 3 = 0$$
$$(2u-1)(u+3) = 0$$
$$2u - 1 = 0 \quad \text{or} \quad u + 3 = 0$$
$$u = \frac{1}{2} \quad \text{or} \quad u = -3$$
$$\cot x = \frac{1}{2} \quad \text{or} \quad \cot x = -3$$
$$\therefore \tan x = 2 \quad \text{or} \quad \tan x = -\frac{1}{3}$$

Many students had difficulty with this. Since the equation results with $\tan x = \ldots$ it would be useful to rewrite $\text{cosec}^2\theta$ as $1+\cot^2\theta$. Students are encouraged to write the 2 identities not given ($1+\cot^2\theta = \text{cosec}^2\theta$ and $\tan^2\theta + 1 = \sec^2\theta$) on the reference sheet before they start the examination.

(ii)	Hence, or otherwise solve $2\,\text{cosec}^2 x = 5 - 5\cot x$ in the domain $-\pi \le x \le \pi$ giving your answers correct to 1 decimal place.	**2**

$$2\,\text{cosec}^2 x = 5 - 5\cot x$$

From (i) $\tan x = 2$ or $\tan x = -\frac{1}{3}$

$\tan x = 2$

Related angle $= 1.1$

$x = 1.1,\ (-\pi + 1.1)$

$= 1.1,\ -2.0$

$\tan x = -\frac{1}{3}$

Related angle $= 0.3$

$x\ \ -0.3,\ (\pi - 0.3)$

$= -0.3,\ 2.8$

$\therefore$ solutions are $x = -2.0, -0.3, 1.1, 2.8$ correct to 1 d.p.

Many students could not find the angles for the stated domain.

Question 12 (15 marks)

(a) The volume V m^3 of water in a tank at time t seconds is given by **2**

$$V = \frac{1}{2}t^4 - 2t^3 + 3t + 5, \quad \text{for } t \geq 0$$

Find the rate at which the volume is changing after 3 seconds.

$V = \frac{1}{2}t^4 - 2t^3 + 3t + 5$

$\frac{dV}{dt} = 2t^3 - 6t^2 + 3$

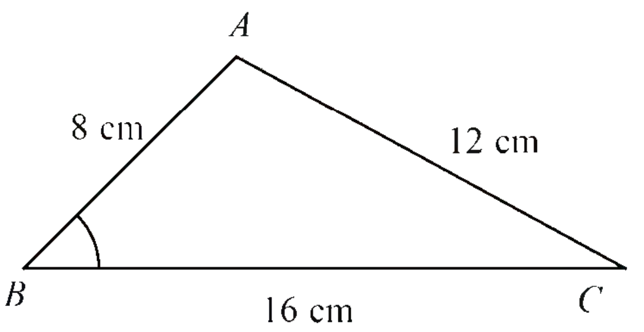
when $t = 3$, $\frac{dv}{dt} = 2 \times 27 - 6 \times 9 + 3$

$= 3$

the volume is increasing at a rate of $3\,m^3/s$.

Very well done. However, some students need to read the units more carefully. It is m^3/s not m^2/s or m/s

(b) The triangle ABC shown below has sides $AB = 8$ cm, $BC = 16$ cm and $AC = 12$ cm respectively.



NOT TO SCALE

(i) Show that $\angle ABC = 46°34'$ correct to the nearest minute. **2**

$\cos A\hat{B}C = \frac{8^2 + 16^2 - 12^2}{2 \times 8 \times 16}$ Cosine Rule

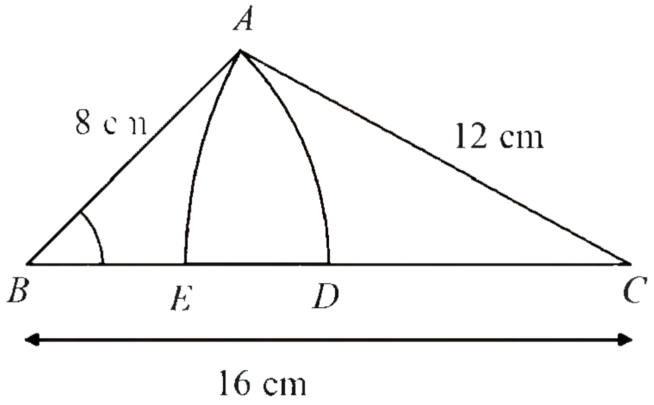
$= \frac{11}{16}$

$A\hat{B}C = 46.5674\ldots$

$= 46°34'$ (nearest minute)

A large number of students received 1 out of 2 for this question. It should emphasised that all working must be shown in show questions. If students did not show the approximation before rounding ie (46.5674 or $46^\circ 34' 2.87''$) they only received 1 mark.

Circular arcs are now drawn from B and C through A to meet BC at D and E respectively.



NOT TO SCALE

(ii) Show that the length of arc AD is 6.5 cm. **1**

(ii) arc length $= r\theta$

$\theta = 46° 34' \times \frac{\pi}{180}$

$= 0.8127\ldots$ (in radians)

length arc $AD = 8 \times 0.8127.$

$= 6.50\ldots$

$= 6.5$ cm

Generally well done. Again as it is a show question, all working was needed.

(iii) Given that $\angle ACB = 0.505$ (3 s.f.), find the area of sector ACE and sector ABD. **2**

Area of sector ACE $= \frac{1}{2} \times 12^2 \times 0.505$

$= 36.36$ cm^2

Area of sector ABD $= \frac{1}{2} \times 8^2 \times 0.8127.$

$= 26.01$ cm^2 (2dp)

Very well done.

(iv) Hence, or otherwise, find the shaded area.	1

Shaded area = Area sector ACE + Area sector AED − Area $\triangle ABC$

$= 36.36 + 26.01 - \frac{1}{2} \times 8 \times 16 \times \sin 46.3^\circ$

$= 36.36 + 26.01 - 46.475..$

$= 15.8948... = 15.89\ cm^2$ (2dp).

Not well done. Students who recognised that they needed to use and calculate the area of the triangle received ½ mark, however a large number of students just took the area of the larger sector and subtracted the smaller sector.

(c) Consider the arithmetic sequence 1000, 994, 988, 982,…. Find the maximum value of S_n, the sum of the first n terms.	3

1000, 994, 988, 982, ...

$a = 1000 \quad d = -6$

Note that the sum increases as long as the terms are positive.

$T_n > 0 \Rightarrow 1000 + (n-1)(-6) > 0$

$1006 - 6n > 0$

$6n < 1000$

$n < \frac{1000}{6}$

$n < 166.66$

T_{166} is the last +ve term $\therefore S_{166}$ is the max. sum

ALTERNATELY

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2000 + (n-1)(-6)]$

$= \frac{n}{2}[2000 + 6 - 6n]$

$= \frac{n}{2}[2006 - 6n]$

$S_n = n[1003 - 3n] = 1003n - 3n^2$

This is a quadratic expression. Maximum occurs at the vertex.

vertex is at $n = \frac{-1003}{2(-3)} = 167.166\ldots$

$S_{167} = 83834$ $\qquad S_{168} = 83832$

$\therefore S_{max} = 83834$

Students who found the smallest positive term here were generally more successful than students who used other methods. Many students decided to find the maximum value of the quadratic expression for the sum. If students used this method they needed to test both S_{167} and S_{168} to see which one gives the greater sum. A significant number of students found that the maximum occurred when $n = \frac{1003}{6}$ and then proceeded to substitute this into the formula for the sum which resulted in a decimal which they then rounded. Students should be reminded that n must be an **integer** value and that when adding integers, they should expect an integer solution, not a decimal.

(d) The rate at which the concentration of a drug in a patient's bloodstream is decreasing is proportional to the concentration of the drug C present in the bloodstream at that time. The time is measured in hours, from the time the drug was administered and C is the concentration of the drug in micrograms per litre.

The concentration of the drug can be described by the model $C = C_0 e^{-kt}$, where C_0 is the initial concentration and k is a positive constant based on the age and weight of the patient.

(i) In a particular patient, it was observed that the drug concentration was reduced to 40% of its initial value after two hours. **2**

Show that the exact value of k is $\frac{1}{2}\log_e\left(\frac{5}{2}\right)$.

$C = C_0 e^{-kt}$

when $t = 2 \qquad C = 0.4\,C_0$

$0.4\,\cancel{C_0} = \cancel{C_0}\, e^{-2k}$

$e^{-2k} = 0.4$

Taking natural logs

$-2k = \ln 0.4$

$k = -\frac{1}{2}\ln 0.4$

$k = -\frac{1}{2}\ln\frac{4}{10}$

$= \frac{1}{2}\ln\left(\frac{10}{4}\right) \qquad \left[\because -\ln x = \ln\left(\frac{1}{x}\right)\right.$

$= \frac{1}{2}\ln\left(\frac{5}{2}\right)$

Overall well done. However, some students lost ½ mark here for not showing enough working to get to the answer of $k = \frac{1}{2}\log_e\frac{5}{2}$ particularly as this value was given to them.

(ii)	The first dose was administered to the patient at midnight. The next dose is to be given when the concentration drops to 10% of its initial value. At what time should the patient be given the next dose?	**2**

(d) (ii) $0.1\cancel{C_0} = \cancel{C_0}e^{-kt}$ where $k = \frac{1}{2}\ln\left(\frac{5}{2}\right)$

$e^{-kt} = 0.1$

$-kt = \ln 0.1$

$t = \frac{\ln 0.1}{-k}$

$= 5.025\ldots$

$=$ 5hr 1 min

$\therefore$ the next dose should be given at 5:01 am.

Very well done.

Question 13 (15 marks)

(a)	Find the values of k for which the function $f(x) = 2x^3 - kx^2 + x + 1$ is increasing for all values of x.	**3**

(a) $f(x) = 2x^3 - kx^2 + x + 1$

$f'(x) = 6x^2 - 2kx + 1$

If $f(x)$ is increasing for all values of x, then $f'(x) > 0$ for all values of x

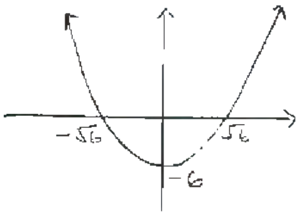
ie. $f'(x) = 6x^2 - 2kx + 1$ is positive definite

$\therefore \quad \Delta < 0$

$\Delta = (-2k)^2 - 4(6)(1) < 0$

$4k^2 - 24 < 0$

$k^2 - 6 < 0$



$-\sqrt{6} < k < \sqrt{6}$

$\therefore$ $f(x)$ is increasing for all values of x for $-\sqrt{6} < k < \sqrt{6}$.

Although students knew that for an increasing function $f'(x) > 0$ they had difficulty applying it to the function. Furthermore, many students who could write an inequality couldn't successfully solve it.

(b)	The limiting sum of an infinite geometric series is five times the first term.	
	(i) Show that the common ratio of the series is 0.8.	**2**

(i) $S_\infty = \dfrac{a}{1-r}$

$\dfrac{\cancel{a}\,1}{1-r} = 5\cancel{a} \qquad (a \neq 0)$

$5(1-r) = 1$

$1 - r = \dfrac{1}{5}$

$r = 1 - \dfrac{1}{5} = \dfrac{4}{5}$

$r = 0.8$

(ii)	If the first term is 20, find the least value of n for which the n^{th} term is less than 1.	**2**

(ii) $a = 20$

$T_n = ar^{n-1}$

$= 20\,(0.8)^{n-1}$

$T_n < 1 \Rightarrow 20 \times (0.8)^{n-1} < 1$

$(0.8)^{n-1} < \frac{1}{20}$

$(n-1)\log(0.8) < \log(0.05)$

$n-1 > \frac{\log(0.05)}{\log(0.8)}$

$n-1 > 13.425$

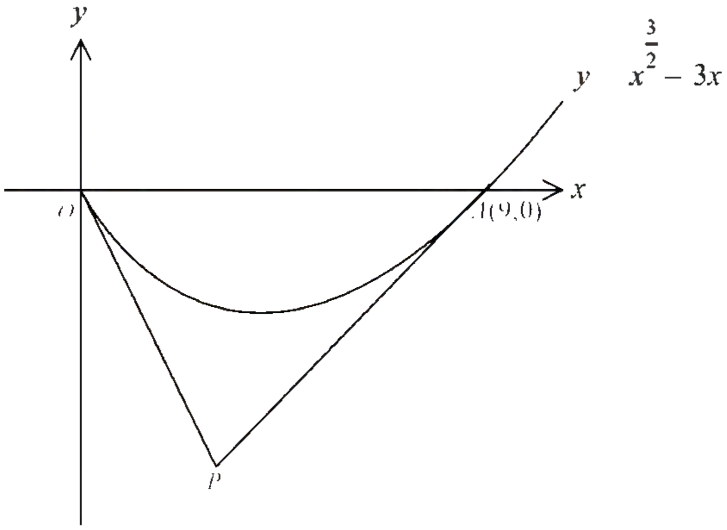
$n > 14.425$

The least value of n for which $T_n<1$ is $n=15$.

Most students could write Tn <1 and made progress. The most common error was not changing the inequality sign when dividing by log0.8.Some students had the correct answer with incorrect working, which was penalised.

(c) A portion of the curve $y = x^{\frac{3}{2}} - 3x$ for $x \geq 0$ is shown below.

The curve passes through the origin and cuts the positive x-axis again at $A(9,0)$.



(i)	Find $\frac{dy}{dx}$.	**1**

(i) $y = x^{3/2} - 3x$

$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} - 3.$

(ii)	Show that the tangent to the curve at A is $3x-2y-27=0$.	**2**

At $A(9,0)$ $\frac{dy}{dx} = \frac{3}{2} \times 9^{\frac{1}{2}} - 3$

$= \frac{9}{2} - 3 = \frac{3}{2}$

$\therefore$ gradient of tangent at $A(9,0)$ is $\frac{3}{2}$

Equation of tangent is $y-0 = \frac{3}{2}(x-9)$

$2y = 3x - 27$

$3x - 2y - 27 = 0$

(iii)	It is given that the equation of the tangent to the curve at O is $3x+y=0$. Find the coordinates of the point P where the two tangents intersect.	**2**

Tangents are $3x - 2y - 27 = 0$ (1)

$3x + y = 0$ (2)

Solving simultaneously for point of intersection

(1) − (2) : $-3y - 27 = 0$

$-3y = 27$

$y = -9$

Sub $y = -9$ in (2)

$3x - 9 = 0$

$3x = 9$

$x = 3$

$\therefore$ $P = (3, -9)$

(iv)	Find the area of the shaded region enclosed between the curve and the two tangents.	**3**

(iv) Shaded area = Area ΔOPA − Area betn curve & x-axis

Area ΔOPA = $\frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times 9 \times 9$

$= \frac{81}{2}$

Area betn curve & axis $= -\int_0^9 (x^{3/2} - 3x)\,dx$

$$= -\left[\frac{2}{5}x^{5/2} - \frac{3}{2}x^2\right]_0^9$$

$$= -\left[\left(\frac{2}{5}\times 9^{5/2} - \frac{3}{2}\times 9^2\right) - 0\right]$$

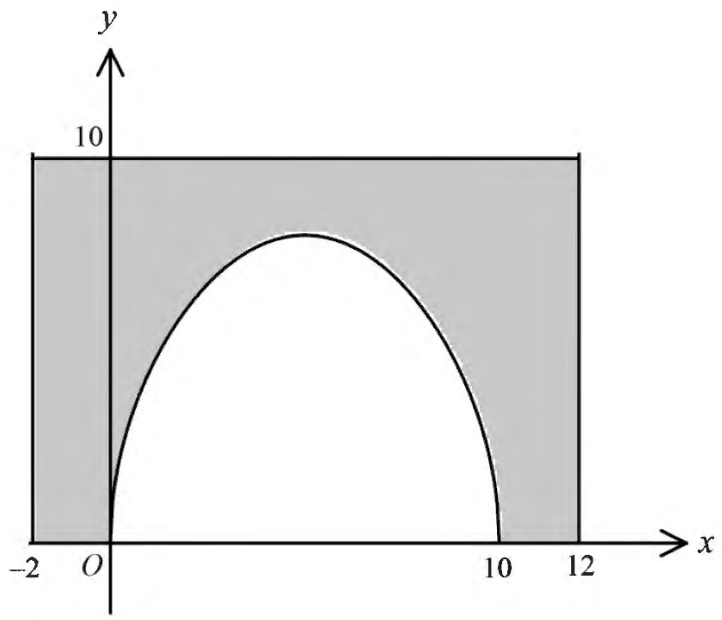
$$= \frac{243}{10}$$

$\therefore$ Shaded area $= \frac{81}{2} - \frac{243}{10} = \frac{81}{5}$ or 16.2 units^2

The last part of this question was not well done. Students who are using the longer approach of "area between two curves" need to write 2 integrals, no absolute value signs are required. For students using simple method, as per solutions, had greater success but needed to realise that the area between the curve and the *x*-axis was negative, and needed to change it to positive.

Question 14 (15 marks)

(a) The cross-section of a tunnel with its concrete surround is shown below.



The concrete surround is represented by the shaded region bounded by the curve, the lines $x = -2,\ x = 12,\ y = 10$ and the x-axis. The units on both axes are in metres.

The height of the tunnel is measured at regular intervals. The results are shown in the table below correct to two decimal places.

x	0	2	4	6	8	10
y	0	6.13	7.80	7.80	6.13	0

(i) The area of the cross-section of the tunnel is given by $\int_0^{10} y\, dx$. **2**

Estimate this area using the Trapezoidal Rule with the given function values.

x	y	wt.	$y \times$ wt
0	0	1	0
2	6.13	2	12.26
4	7.80	2	15.60
6	7.80	2	15.60
8	6.13	2	12.26
10	0	1	0

Total = $\underline{55.72}$

Area cross section $= \int_0^{10} y\, dx$

$\doteqdot \frac{2}{2} \times 55.72$

$= 55.72 \ m^2$

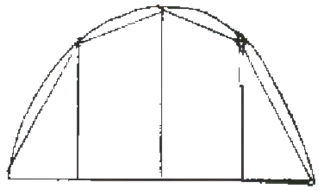
(ii)	Hence, deduce an estimate for the cross-section area of the concrete surround.	**1**

Cross section of concrete $\doteqdot 14 \times 10 - 55.72$

$\doteqdot 84.28 \ m^2$

(iii)	State with justification whether the area of the concrete cross-section found in (ii) is an over-estimate or an underestimate.	**1**

(iv)



As the tunnel is concave down, the Trapezoidal Rule underestimates the cross-section of the tunnel, as shown in the diagram. Therefore, when we subtract this from the area of the rectangle, the resulting estimate for the concrete cross-section is an over-estimate.

i. This question was done well. Most students were able to recall the trapezoidal rule correctly to estimate the cross-section of the tunnel.
ii. This question was done well. Students need to use the answer obtained in i.
iii There were numerous mistakes that led to students not being awarded the full mark for this question. Students need to read the question carefully as the question asked for the area of the concrete cross-section, not the tunnel cross-section. Students should either provide a diagram as part of their justification or explain that since the tunnel is concave down, the estimation of the tunnel using trapezoidal rule is an underestimate. Hence the concrete cross-section found in ii. is an over-estimate. Students need to be mindful that a trapezoidal rule does not always underestimate the integral being evaluated.

(b)	Consider the function $f(x) = x^4 - 3x^3 + 3$.	
(i)	Find the stationary points on the curve and determine their nature.	**3**

(b) $f(x) = x^4 - 3x^3 + 3$

(i) $f'(x) = 4x^3 - 9x^2$

$= x^2(4x - 9)$

At stationary points, $f'(x) = 0$

$x^2(4x-9) = 0$

$x = 0, \ \frac{9}{4}$

$f(0) = 3 \qquad f\left(\frac{9}{4}\right) = -\frac{1419}{256}$

$\therefore$ stationary points are at $(0,3)$ and $\left(\frac{9}{4}, -\frac{1419}{256}\right.$

$f''(x) = 12x^2 - 18x$

$f''\left(\frac{9}{4}\right) = 20.25 > 0$

So the curve is concave up at $x = \frac{9}{4}$ and there is a minimum turning point at $\left(\frac{9}{4}, -\frac{1417}{256}\right)$

$f''(0) = 0$ so examine $f'(x)$ on either side of $x = 0$

x	-1	0	1
$f'(x)$	-13	0	-5
	<0		<0

$\therefore$ At $(0,3)$ there is a horizontal point of inflexion

(ii)	Find the x-coordinates of any points of inflexion.	**2**

(ii) At inflexion points $f''(x) = 0$

$$12x^2 - 18x = 0$$
$$6x(2x - 3) = 0$$
$$x = 0, \frac{3}{2}$$

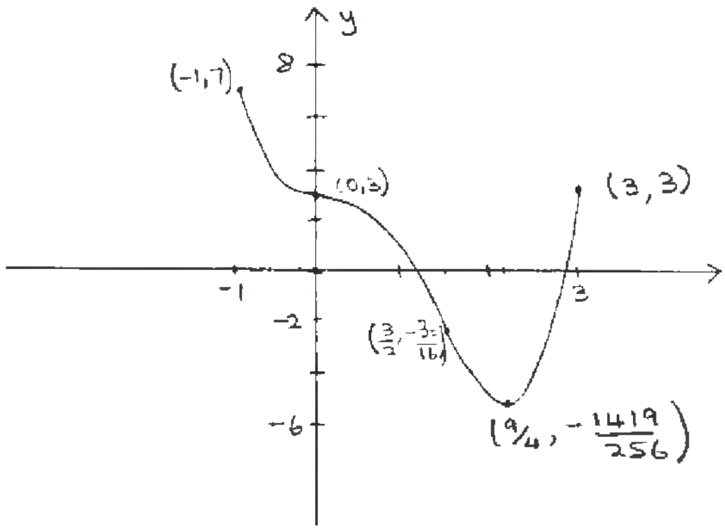
At $x = 0$, there is a horizontal point of inflexion (shown in part (i))

Check for change in concavity at $x = \frac{3}{2}$

x	1	$\frac{3}{2}$	2
$f''(x)$	-6	0	12
	<0		>0

There is a change in concavity, so there is a point of inflexion at $x = \frac{3}{2}$.

(iii)	Sketch the curve for $-1 \le x \le 3$ showing all the information from parts (i) and (ii). You are not required to find the x-intercepts.	**2**

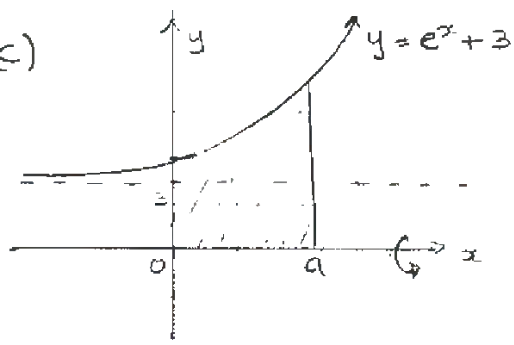


Overall, this part was not done well. Students should be familiar with this type of question, as they are commonly asked and the process in determining the answers is fairly straightforward.

i. Most students were able to obtain the first derivative correctly and hence determined the x coordinate of the stationary points correctly. The first derivative and second derivative tests can then be used to determine the nature of the stationary points. However, when using the second derivative test to determine the nature of the stationary point at $x = 0$, the result yields a POTENTIAL horizontal point of inflexion. Students need to check for a change in concavity or refer to the gradient before and after the stationary point to safely conclude that it is a horizontal point of inflexion. Marks were deducted for inconclusively determining the nature of the stationary points.
ii. Similar to i., most students were able to obtain the correct second derivative and therefore correctly determined the POTENTIAL point of inflexions. Students need to show that there is a change in concavity about these points for them to be true point of inflexions.
iii. Several mistakes were common across all students. Firstly, students need to be aware of the domain restrictions and therefore the end points need to be determined and indicated on the graph. Secondly, at the horizontal point of inflexion, a stationary point needs to be shown. Lastly, the point of inflexion found in ii. as well as the minimum turning point found in i. need to be indicated on the graph.

(c)	The region enclosed between the curve $y = e^x + 3$ and the x-axis from $x = 0$ to $x = a$ is rotated about the x-axis to form a solid of revolution. If the volume of this solid is $\pi(9a + 16)$ units^3, find the exact value of a.	**4**

(c)



$$V = \pi \int_0^a y^2\,dx \qquad y = e^x + 3$$
$$y^2 = e^{2x} + 6e^x + 9$$

$$= \pi \int_0^a \left(e^{2x} + 6e^x + 9\right) dx$$

$$= \pi \left[\frac{e^{2x}}{2} + 6e^x + 9x\right]_0^a$$

$$= \pi \left[\left(\frac{e^{2a}}{2} + 6e^a + 9a\right) - \left(\frac{1}{2} + 6 + 0\right)\right]$$

$$= \pi \left(\frac{e^{2a}}{2} + 6e^a + 9a - \frac{13}{2}\right)$$

Now $\cancel{\pi}\left(\frac{e^{2a}}{2} + 6e^a + 9a - \frac{13}{2}\right) = \cancel{\pi}(9a + 16)$

$$\frac{e^{2a}}{2} + 6e^a + \cancel{9a} - \frac{13}{2} = \cancel{9a} + 16$$

$$e^{2a} + 12e^a - 13 = 32$$
$$e^{2a} + 12e^a - 45 = 0$$

Let $u = e^a$ $\qquad u^2 + 12u - 45 = 0$

$$(u + 15)(u - 3) = 0$$
$$u = -15 \quad \text{or} \quad u = 3$$
$$e^a = -15 \quad \text{or} \quad e^a = 3$$
no solutions $\qquad a = \ln 3$

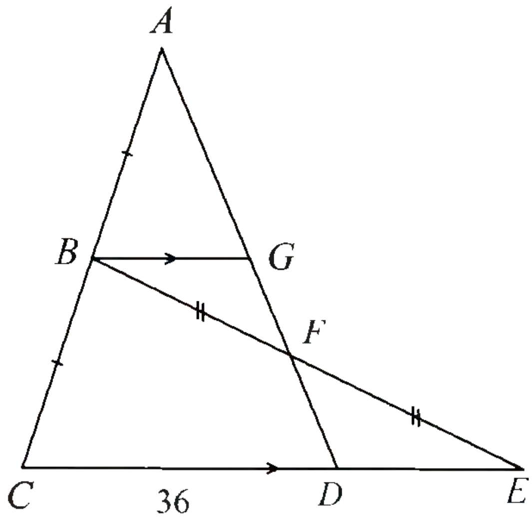
$\therefore \quad a = \ln 3$

This part was generally well done. Most candidates were able to recognise that volume of the solid generated can be determined by $\pi\int_0^a y^2 dx$. Students did well in evaluating the integral and then equating that to the volume given.

The result is an equation reducible to quadratics and most students were able to solve the equation to obtain a solution for a. Students who made algebraic mistakes along the way were generally unsuccessful as an equation reducible to quadratics cannot be obtained.

Question 15 (15 marks)

(a) In the diagram below, $AB = BC$ and $EF = FB$ as shown. BG is parallel to CE and $CD = 36$ cm.



(i) Prove that ΔDEF and ΔBGF are congruent. **3**

In ΔDEF and ΔBGF

$BF = FE$ (given)

$\angle BFG = \angle EFD$ (vertically opposite)

$\angle GBF = \angle FED$ (alternate angles, $BG \parallel CE$)

$\therefore \Delta BFG \equiv \Delta EFD$ (AAS)

Mostly well done. A few students did a similarity proof.

(ii) Hence, or otherwise, find the length of DE. **2**

$BG = \frac{1}{2} CD$ (converse of join of midpoints)

$= \frac{1}{2} \times 36$

$= 18$

$BG = DE$ (matching sides of congruent triangles)

$\therefore DE = 18$

Most students got DE =18cm but with wrong reasoning. Most common incorrect reason was "Parallel lines preserve ratio" instead of "Converse of Midpoint theorem".
Alternate answer: To find the length of DE, some students used the ratio of matching sides of similar triangles after proving triangle *ABG* similar to triangle *ACD*.

(iii)	Given that $\angle CAD = \angle GBF$, prove that ΔACD and ΔEBC are similar.	**2**

$\angle CAD = \angle GBF$ (given)

$\angle GBF = \angle FED$ proved above in (i)

$\therefore \angle CAD = \angle FED$

$= \angle CEB$ (same angle)

In ΔCAD and ΔCEB

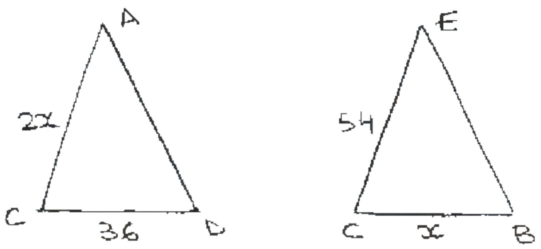
$\angle ACD = \angle ECB$ (shared angle)

$\angle CAD = \angle CEB$ (proved above)

$\therefore \Delta CAD \parallel\!\!| \Delta CEB$ (equiangular)

(iv)	Deduce the exact length of AC.	**2**

(iv)



Let $BC = x$

Then $AC = 2x$ (E is the midpoint of AC)

$CE = CD + DE$

$= 36 + 18$

$= 54$

$\frac{AC}{CE} = \frac{CD}{BC}$ (matching sides of similar triangles)

$\frac{2x}{54} = \frac{36}{x}$

$2x^2 = 36 \times 54$

$x^2 = \frac{36 \times \cancel{54}^{27}}{\cancel{2}}$

$x = 6 \times 3\sqrt{3}$ $\qquad x > 0$

$= 18\sqrt{3}$

$\therefore AC = 2x = 36\sqrt{3}$

Many students did not successfully answer this question. Some students got the ratio of matching sides wrong and few students assumed $CE = BE$ to find the length of AC.

(b) The motion of a particle moving in a straight line is described by $v = \dfrac{6}{t+3}$, where v is the velocity in metres/second and t is the time in seconds. Initially the particle is at the origin.

(i) What is the acceleration of the particle 2 seconds after the start of the motion? **2**

(b) $v = \dfrac{6}{t+3} = 6(t+3)^{-1}$

(i) $a = \dfrac{dv}{dt}$

$= 6 \times -1 \, (t+3)^{-2}$

$= \dfrac{-6}{(t+3)^2}$

At $t = 2$, $a = \dfrac{-6}{5^2} = \dfrac{-6}{25}$ m/s^2

(ii) Find the displacement, x, as a function of time. **2**

(ii) $x = \int v \, dt$

$= \int \dfrac{6}{t+3} \, dt$

$x = 6 \ln(t+3) + C$

At $t = 0$, $x = 0$

$0 = 6 \ln(3) + C$

$\therefore C = -6 \ln 3$

$x = 6 \ln(t+3) - 6 \ln 3$

$x = 6 \ln \left(\dfrac{t+3}{3}\right)$

(iii) Briefly describe the motion of the particle. **2**

Initially, the particle is at the origin, moving to the right with a velocity of 2 m/s. As the velocity is always positive and acceleration is always negative, the particle is always moving to the right and slowing down.

Many students did not successfully answer this question. To get full marks, the response needed to include "the particle moves to the right" and "it slows down" (or velocity decreases).

Question 16 (15 marks)

(a) (i) Given that $f(x) = x^4 + 2x$, find $f'(x)$. **1**

$f(x) = x^4 + 2x$

$f'(x) = 4x^3 + 2$

(ii) Hence, or otherwise, find $\int_1^3 \frac{2x^3+1}{x^4+2x}\,dx$. **2**

$$\int_1^3 \frac{2x^3+1}{x^4+2x}\,dx = \frac{1}{2}\int_1^3 \frac{4x^3+2}{x^4+2x}\,dx$$

$$= \frac{1}{2}\left[\ln(x^4+2x)\right]_1^3 \quad \text{using (i)}$$

$$= \frac{1}{2}\left[\ln(87) - \ln(3)\right]$$

$$= \frac{1}{2}\ln\left(\frac{87}{3}\right)$$

Generally very well done.

(b) Selina borrowed an amount $\$P$ at a rate of 3.6% per annum compounded monthly on a reducing balance. At the end of each month, she repays an amount of $\$M$.

(i) Show that the amount owing at the end of the second month is **1**

$A_2 = P(1.003)^2 - M(1.003) - M$.

(b) rate $= 3.6\%$ p.a

$= \frac{3.6}{12}\%$ per month

$= 0.003$

(i) $A_1 = P \times 1.003 - M$

$A_2 = A_1 \times 1.003 - M$

$= (P \times 1.003 - M) \times 1.003 - M$

$= P \times (1.003)^2 - M \times 1.003 - M$ as req,d

(ii)	It is now given that the loan amount is \$50 000 and her monthly repayment is \$350. Starting with your result in (i), show that the amount owing after one year is \$ 47 560.	**2**

$$A_3 = A_2 \times 1.003 - M$$

$$= P \times 1.003^3 - M \times 1.003^2 - M \times 1.003 - M$$
$$= P \times 1.003^3 - M\left(1 + 1.003 + 1.003^2\right)$$

$$A_n = P \times 1.003^n - M\left(1 + 1.003 + 1.003^2 + \dots + 1.003^{n-1}\right)$$

Now, $P = 50000$ $\quad M = 350$

$$A_{12} = 50000 \times 1.003^{12} - 350\,\underbrace{\left(1 + 1.003 + 1.003^2 + \dots + 1.003^{11}\right)}_{\text{This is a geometric series } a=1,\ r=1.003,\ n=12}$$

$$A_{12} = 50000 \times 1.003^{12} - 350\,\frac{\left(1.003^{12} - 1\right)}{1.003 - 1}$$ using sum of geometric series.

$$= 47560.00131$$
$$= 47560$$

$\therefore$ amount outstanding after 1 year is \$ 47560

(iii)	Find how many years it will take Selina to repay the loan fully.	**3**

$$A_n = 50000 \times 1.003^n - 350 \times \frac{\left(1.003^n - 1\right)}{0.003}$$

When the loan is repaid $A_n = 0$

$$50000 \times 1.003^n - 350 \times \frac{1.003^n - 1}{0.003} = 0$$

$$\left(50000 - \frac{350}{0.003}\right) \times 1.003^n + \frac{350}{0.003} = 0$$

$$-\frac{200000}{3} \times 1.003^n = -\frac{350000}{3}$$

Multiply both sides by $-\frac{3}{10000}$

$$20 \times 1.003^n = 35$$
$$1.003^n = \frac{35}{20} = \frac{7}{4}$$

$$n \log 1.003 = \log\left(\frac{7}{4}\right)$$

$n = \log\left(\frac{7}{4}\right) \div \log 1.003$

$= 186.818..$ months

$= 15.568...$ years

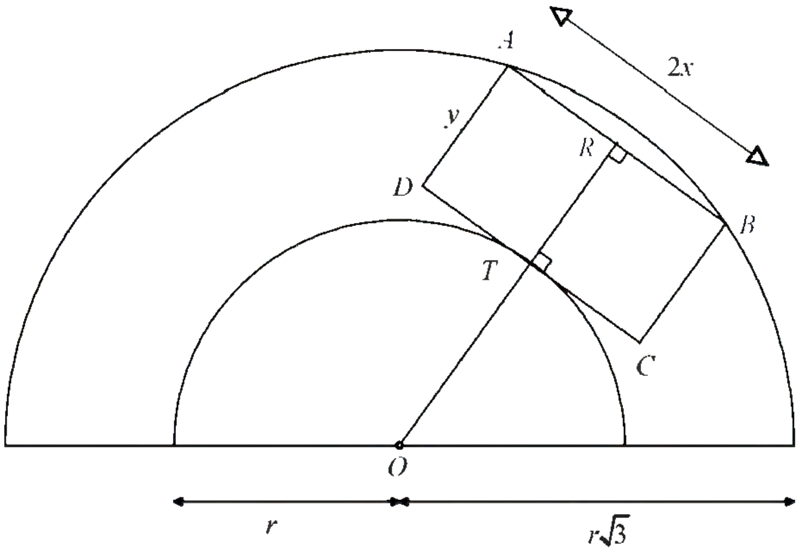
The loan is fully repaid after 15 years 7 months

Generally well done.

(c) A rectangular box is required to be moved along a corridor bounded by curved vertical walls. A cross-section of the corridor is shown below.

The curved walls are two concentric semicircles centred at O, of radii r and $r\sqrt{3}$ respectively.

The rectangular box $ABCD$ has length $2x$ and width y as shown. CD is tangential to the inner wall at T, where T is the midpoint of CD. A and B are in contact with the outer wall. OTR is perpendicular to CD and AB.



(i) Show that $x^2 = 2r^2 - 2ry - y^2$. **2**

(c)(i) Join OB.

$\triangle$ ORB is right angled

$OR^2 + RB^2 = OB^2$ (Pythagoras)

Now, $OR = OT + TR$

$= r + y$

$RB = \frac{1}{2} \times AB$

$= x$

$OB = r\sqrt{3}$

$$\therefore \quad (r+y)^2 + x^2 = (r\sqrt{3})^2$$
$$r^2 + y^2 + 2ry + x^2 = 3r^2$$

Rearranging, we get
$$x^2 = 3r^2 - r^2 - 2ry - y^2$$
$$= 2r^2 - 2ry - y^2 \quad \text{as reqd}$$

(ii) Show that the area of *ABCD* is maximum when $y = \dfrac{r}{2}$. **4**

(ii) let Area of ABCD = A

$$A = 2x \times y$$
$$= 2y\sqrt{2r^2 - 2ry - y^2}$$
$$= 2\sqrt{2r^2y^2 - 2ry^3 - y^4}$$
$$= 2\,(2r^2y^2 - 2ry^3 - y^4)^{\frac{1}{2}}$$

$$\frac{dA}{dy} = \cancel{2} \times \frac{1}{\cancel{2}}(2r^2y^2 - 2ry^3 - y^4)^{-\frac{1}{2}} \times (4r^2y - 6ry^2 - 4y^3)$$
$$= \frac{4r^2y - 6ry^2 - 4y^3}{\sqrt{2r^2y^2 - 2ry^3 - y^4}}$$
$$= \frac{2y\,(2r^2 - 3ry - 2y^2)}{\sqrt{2r^2y^2 - 2ry^3 - y^4}}$$

At stationary points $\frac{dA}{dy} = 0$

$$\frac{dA}{dy} = 0 \Rightarrow 2y\,(2r^2 - 3ry - 2y^2) = 0$$

$y = 0$ or $2r^2 - 3ry - 2y^2 = 0$

$y > 0$ (length) so $2r^2 - 3ry - 2y^2 = 0$

$(2r + y)(r - 2y) = 0$

$2r + y = 0$ or $r - 2y = 0$

$y = -2r$ or $y = \frac{r}{2}$

$y > 0$ so $y = \frac{r}{2}$

stationary point at $y = \frac{r}{2}$

Test nature.

sign of $\frac{dA}{dy}$ is the same as the sign of $2r^2-3ry-2y^2$

as denominator $\sqrt{2r^2y^2-2ry^3-y^4}$ is always positive & $y>$

y	$\frac{r}{3}$	$\frac{r}{2}$	$\frac{2r}{3}$
$(2r+y)(r-2y)$	$\frac{7r}{3}\times\frac{r}{3}$ $=\frac{7r^2}{9}$	0	$\frac{8r}{3}\times\frac{-r}{3}$ $=-\frac{8r^2}{9}$
	$\frac{dA}{dy}>0$		$\frac{dA}{dy}<0$
	/	—	\

$\therefore$ A is maximum when $y=\frac{r}{2}$.

Alternately,

A is maximum when A^2 is maximum.

$$A^2=(2xy)^2 = 4x^2y^2$$
$$= 4(2r^2-2ry-y^2)y^2$$
$$= 4(2r^2y^2-2ry^3-y^4)$$

$$\frac{d}{dy}(A^2) = 4(4r^2y-6ry^2-4y^3)$$
$$= 8y(2r^2-3ry-2y^2).$$

$\frac{d}{dy}A^2=0 \Rightarrow y=0$ or $2r^2-3ry-2y^2=0$

$y>0$ so $2r^2-3ry-2y^2=0$

$(2r+y)(r-2y)=0$

$y=-2r$ or $y=\frac{r}{2}$

$y>0$ so $y=\frac{r}{2}$

y	$\frac{r}{3}$	$\frac{r}{2}$	$\frac{2r}{3}$
$\frac{d}{dy}A^2$	$\frac{8r}{3}\times\frac{7r^2}{9}$	0	$\frac{16r}{3}\times-\frac{8r^2}{9}$
	>0		<0
	/	—	\

so, A^2 is max at $y=\frac{r}{2}$

and therefore A is max at $y=\frac{r}{2}$

i) Many students could not prove the relationship between *x* and *y* by using the right triangle.

ii) Most students did not use the chain rule to differentiate *A* but they used the product rule which was longer and harder. To find the nature of the stationary point, students needed to show working with numerical values instead of just + , 0 and –.